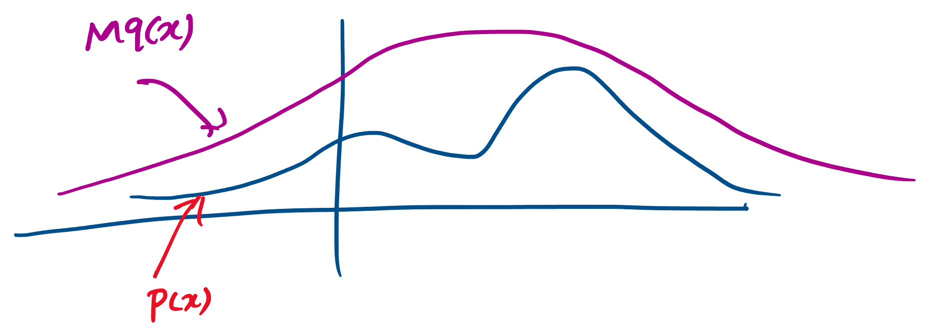
My notes for lec 17

Saturday, March 17, 2018 4:36 PM

Rejection Sampling

- Find $q(x)$ that is easy to sample from
- Find M such that $\frac{\tilde{P}(x)}{M q(x)} \leq 1$



$$P(y=1|x) = \frac{\tilde{p}(x)}{M q(x)}$$ Probability of accepting samples

What's the distribution of the accepted samples?

$$p(x|y=1) = \frac{P(y=1|x)\, q(x)}{P(y=1)} = \frac{\frac{\tilde{p}(x)}{M}}{\int p(y=1|x)\, q(x)\, dx} = \frac{\frac{\tilde{p}(x)}{M}}{\int \frac{\tilde{p}(x)}{M q(x)}\, q(x)\, dx}$$

$$= \frac{\frac{\tilde{p}(x)}{M}}{\frac{1}{M}\int \tilde{p}(x)\, dx} = \frac{\tilde{p}(x)}{Z} = p(x)$$

Why stationary distribution of Gibbs sampling is the distribution?

Given sample x^l at l'th iteration, we draw the next sample from this distribution

$$Q(x \mid x^l) = \sum_i q(i)\, P(x_i \mid x_{-i}^l) \prod_{j \neq i} \delta(x_j, x_j^l)$$

$l+1$ sample (Next sample)

l'th sample

① First choose one of the variables randomly with prob $q(i)$

② Making sure other variables are fixed at the previous iteration

③ Sample from conditional

Let's show the stationary distribution is the same

$$\int_x Q(x'|x)\, p(x)\, dx$$

If I sample from $p(x)$, what's the distribution of the next sample.

$$= \int_x \sum_i q(i)\, p(x_i'|x_{-i}) \prod_{j\neq i} \delta(x_j', x_j)\, \cancel{p(x)}\, dx \quad p(x_i, x_{-i})$$

$$= \sum_i q(i) \int_x \cancel{p(x_i'|x_{-i}) \prod_{j\neq i} \delta(x_j', x_j)\, p(x_i, x_{-i})}\, dx$$

$$p(x_i'|x_{-i}') \qquad p(x_i, x_{-i}')$$

$$= \sum_i q(i) \int_{\cancel{x}\; x_i} p(x_i'|x_{-i}')\, p(x_i, x_{-i}')\, \cancel{dx}\; dx_i$$

$$= \sum_i q(i)\, p(x_i'|x_{-i}') \cancel{\int_{x_i} p(x_i, x_{-i}')\, dx_i}$$

$$P(x_{-i}')$$

$$= \sum_i q(i)\, \cancel{P(x_i'|x_{-i}')\, P(x_{-i}')} = P(x')$$

$$P(x')$$

$P(x)$ is a stationary distribution!